PUSS BANK SCHOOL AND NURSERY

Calculation Policy

Updated September 2015

Introduction

This document is a statement of the aims, principles and strategies for teaching and learning of calculation strategies in Mathematics at Puss Bank Primary School. This policy is shows the end of year calculation expectations for each year group in each operation (addition, subtraction, multiplication and division), and written approaches for solving time problems. All methods should be taught with understanding rather than by rote and put into real life contexts. At the end of the policy is an outline of Puss Bank's approach to solving problems in maths.

Rationale

At Puss Bank Primary School, we believe a clear progression in calculation will support the learning and teaching of maths throughout the school, allow clarity and provide a secure foundation upon which to build and develop mathematical skills.

This policy contains the key mental and written procedures that will be taught within Puss Bank Primary School. Although by the end of year 6 children should be proficient in written methods for all calculations, at the heart of all written methods is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and develop new ideas. Therefore, written recording both helps children to extend and clarify their thinking.

Progression in calculation should include:

 A range of mental strategies to be used as a first resort, even once written methods have been introduced and embedded.
 An ability to understand and use the relationships between the four operations of number.
 An ability to explain, describe and record their methods.
 An ability to estimate and check whether the answer is correct.
 An ability to solve a wide range of problems involving calculation in a wide variety of contexts.
 An ability to choose and use the most appropriate method of calculation; mental, jottings, written or using a calculator.
 An ability to take the initiative to return to an earlier method that children are more confident with.

Children should be encouraged to see mathematics as a mental and spoken language with recording as a written 'record' of these processes. Teachers should support and guide children through the following stages:

Developmental Aims:

 To introduce children to the processes of calculation through practical, oral and mental activities.
 To support children in developing ways of recording to support their thinking and mental calculation methods.
 Enable children to interpret and use the signs and symbols.
 To facilitate children's use of models and images, beginning with concrete images such as Numicon and moving to pictorial images such as the empty number-line.
 To enable children to strengthen and refine their mental methods in order to develop informal written methods.
 To support children in becoming more efficient and succinct in their recordings which will ultimately lead to efficient written methods that can be used more generally.
 By the end of Key Stage 2 children should be equipped with mental, written and calculator methods that they understand and can use correctly.
 By the end of Key Stage 2, when faced with a calculation, children will be able to decide which method is most appropriate and have strategies to check its accuracy.
 At whatever stage in their learning, and whatever method is being used, children's methods of calculating will be underpinned by a secure and a deepen knowledge of number facts, along with the mental skills that are needed to carry out the process and judge if it was successful.

The overall aims when children leave primary school are for them to:

 have a secure understanding of mental maths facts to apply to written mathematics;
 have a secure knowledge of number facts and a **deep** understanding of the four operations
 have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
 be able to use this knowledge and understanding to solve problems;
 use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

Mental methods of calculation

Oral and mental mathematics is essential, particularly so in calculation. Early practical, oral and mental work lays the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later learning and skill development must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied.

Ongoing oral and mental mathematics learning provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills.

Secure mental calculation requires the ability to:

 understand
- the different **structures** of all four operations e.g. to understand subtraction as take away, decrease, difference and the inverse;
- the **relationship** between operations - that subtraction 'undoes' addition, how multiplication and division relate to one another;
- how the **rules and laws** of arithmetic are used and applied - for example, to add or subtract mentally combinations of one-digit and two-digit numbers (Year 3), and to calculate mentally with whole numbers and decimals (Year 6).

 recall key number facts instantly - for example, all addition and subtraction facts for each number to at least 10 (Year 1), sums and differences of multiples of 10 (Year 2)

 recall all times tables up to 12 x 12 by then end of year 4. Learnt as follows:
- **Foundation** – by end of year begin counting sequences
- **Year 1 –** counting sequences (which lays down the foundation for later times tables e.g. counting in multiples of 2 will lead into learning the 2 times table. By the end of year 1, children can begin to use their knowledge of counting in multiples of 2, 5 and 10 to solve simple practical problems involving multiplication and division.
- **Year 2 –** Refine 2, 10, 5 times tables. Learn 3 and 4 times tables.
- **Year 3 –** Refine 2, 10, 5, 3, 4 times tables. Learn 11, 6 and 7 times tables.
- **Year 4-** Refine 2, 10, 5, 3, 4, 11, 6, 7 times tables. Learn 8, 9, 12 times tables.
- **Year 5/6** – continue practice of all times tables up to 12 x 12, use these to inform division and to work out other times tables higher than 12 (e.g. double 12 times tables to generate 24 times tables).

 use taught strategies to work out the calculation - for example, recognise that addition can be done in any order and use this to add mental digit number or a multiple of 10 to a one-digit or two-digit num1), partition two-digit numbers in different ways including into ten and one and add the tens and ones separately and then recombine (Year 2), when applying mental methods in special cases (Year 5) .

The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient written method for each operation with confidence and understanding. Children will develop the ability to use what are commonly known as 'standard' written methods - methods that are efficient and work for any, including those that involve whole numbers or decimals. They compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or through access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

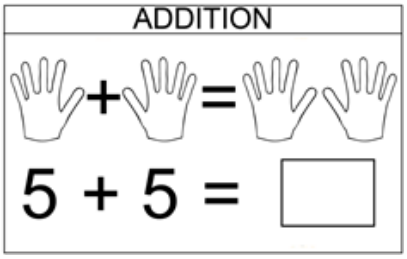
In setting out these aims, the intention is that there will be a consistent approach to the learning of calculation strategies and that all teachers understand the progression of skills and key concepts. The great majority of children will benefit learning how to use the most efficient methods. The challenge for teachers will be in determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation. Guidance is given below for the steps in reaching the most efficient methods for four number operations.

Mathematical Language

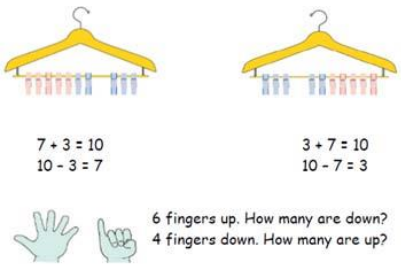
For all calculations we need children to practically carry out the calculation (action), say what they have done (language) and then record this. Therefore, it is extremely important that children are introduced to the mathematical language and practise using this to explain their actions, thinking and the reasoning behind the strategy they have used.

Before children move on to formal methods, which are outlined in the policy, the recoding should always be children's own concept to begin with. Children will need to use the mathematical language to explain their workings and understanding.

Addition
Add
And
Plus
Increase
More than
Total
Altogether
Sum of



Subtraction
Take away
Minus
Decrease
Less than
Difference
How much more
Subtract
Left

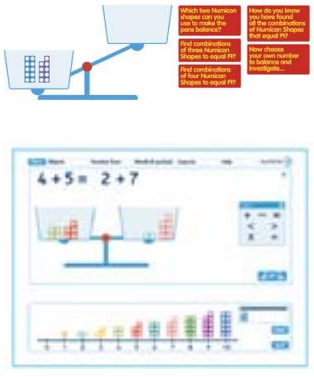


Multiply
Sets of
Lots of
Groups of
Times
Multiply by
Double – Twice, three times etc
Product
Multiple
Factor
Repeated addition

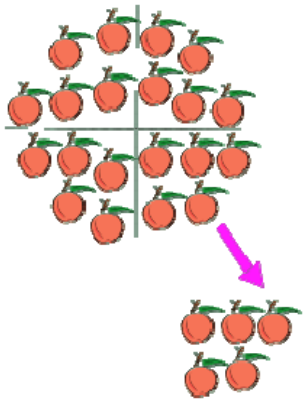
X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Equals
Is the value same as
Is equivalent to

=



Division
Share equally
Divide by
Group
Halve, quarter etc
Chunk
Multiple
Pairs
Remainders
Array



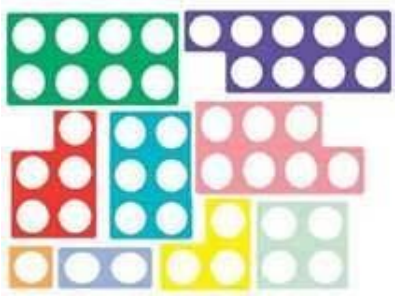
Progression from Mental Methods to Written Methods for Addition

To add successfully, children need to be able to:

- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

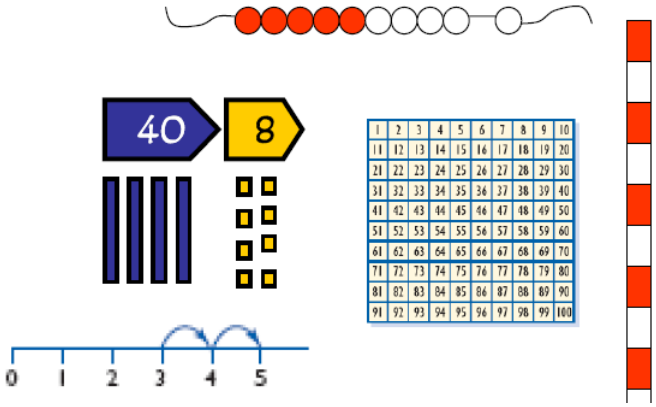
Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Mental Skills



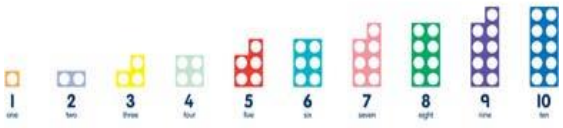
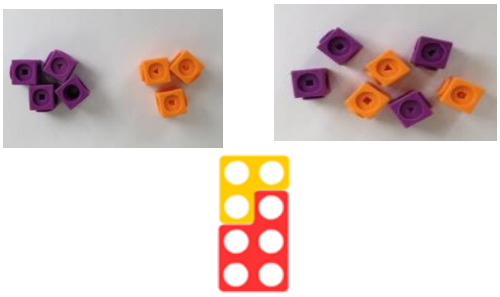
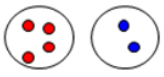
Recognise the size and position of numbers
Count on in ones and tens
Know number bonds to 10 and 20
Add multiples of 10 to any number
Partition and recombine numbers
Bridge through 10
Doubles and near doubles
Building on known facts

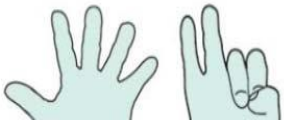
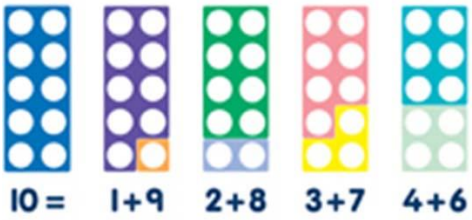
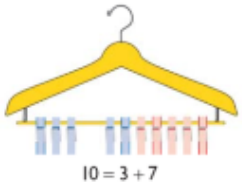
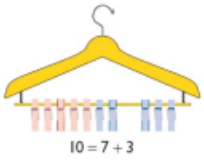
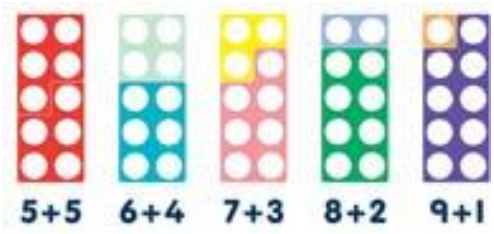
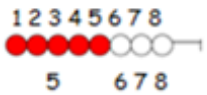
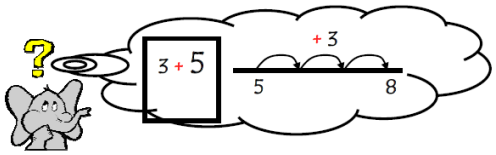
Models and Images

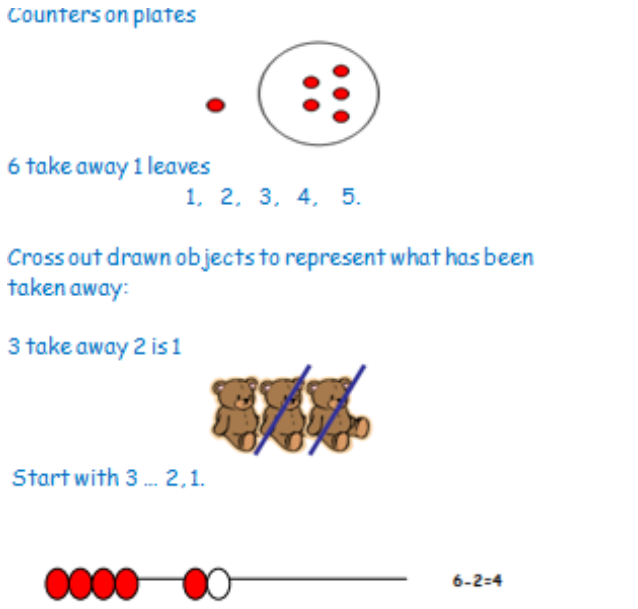
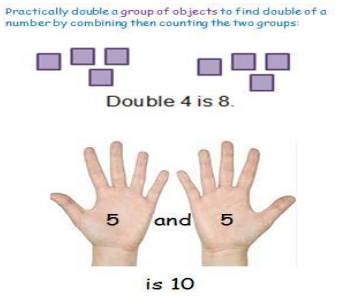


Counting apparatus
Place value apparatus
Place value cards
Number tracks
Numbered number lines
Marked but unnumbered number lines
Empty number lines
Hundred square
Counting stick
Bead string
Numicon
Dienes/Base 10

EYFS

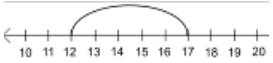
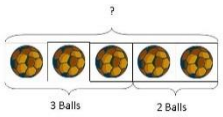
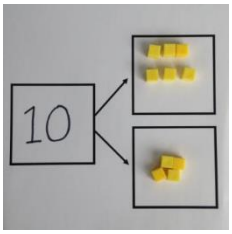
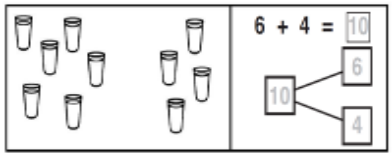
Progression	**Notes**	**Model/ Image**
Children to recognise and represent numbers		Large number tiles/lines displayed Fingers Number track Jottings/Tally Number formation Bead strings
Children will say 1 more or 1 less from a given number (to 10 then to 20)	Children may make up their own number stories/books. Songs and rhymes	Concrete objects e.g. fingers, children cubes. Real life situations Numicon  Number tracks Bead strings Pictorial drawing of amounts and adding on or crossing off using songs rhymes and stories.
ADDITION Children need to be able to **count on** from any number, combining two groups. **6** and **2** more makes **8** **6 + 2 = 8**	Numicon shapes are introduced straight away and can be used to: □ identify 1 more/less □ combine pieces to add. □ find number bonds. □ Add without counting. Children can record this by printing or drawing around Numicon pieces. They may develop ways of recording calculations using pictures or use apparatus, such as Numicon to show this.	Pictorial drawing of amounts using songs rhymes and stories. Multilink using number stories Unifix  5 plus 3 = 8 3 more than 5 =8  1, 2, 3, 4, 5, 6.

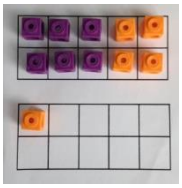
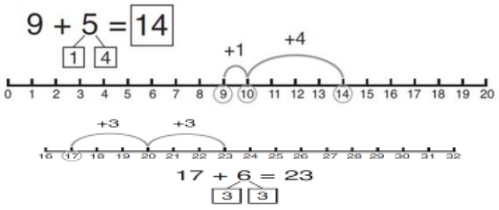
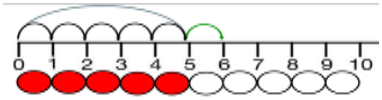
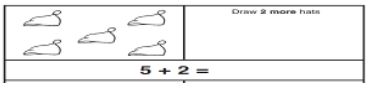
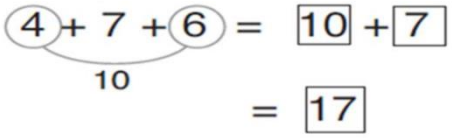
Children will begin to know addition facts to ten.	Numicon is used as a primary resource to support the teaching of addition where children are encouraged to visualise the Numicon patterns and calculate by combining/ partitioning the patterns rather than resort to counting. Practical experiences and activities are at the heart of developing mathematical concepts. Children are encouraged to record in a way that makes sense to them and they can use to explain their understanding to others. Classroom displays should reflect their developing skills and act as prompts.	7 fingers up How many are down?  They will recognise how they can use their fingers to help them with number bond facts. 
Children will recognise that addition can be done in any order. **Children will be able to use more efficient jumps, starting with the larger number and counting on.**	They will begin by counting from zero, and once they become more confident with counting they will understand and be able to count on from the first number. They will count on from the larger number. They will be able to record their calculation as a number sentence. What is ? more than ? If ready, some children may be able to write a number sentence. Number bonds up to 10 recorded through number sentences e.g. 3+3=6, 2+4=6	   5 + 3 = 8  **Put the biggest number first and count on.** 

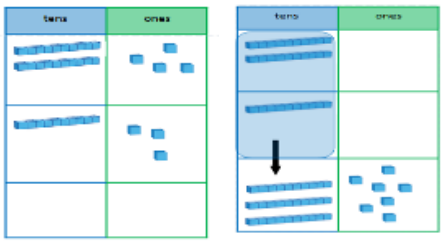
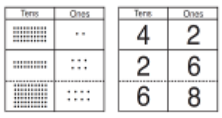
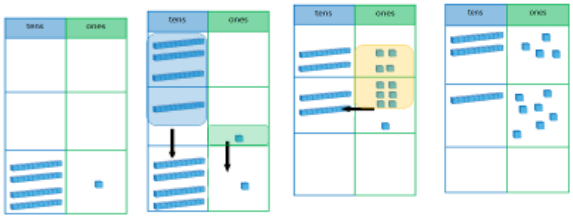
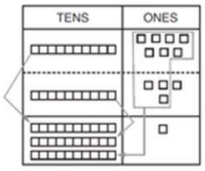
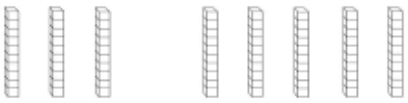
SUBTRACTION **Subtract two single digit numbers**	Children should use a range of manipulatives, including counters and bead strings to support calculation. Physical jumping on a number track, will support progression to number lines when understanding is secure. Children should be encouraged to imagine a number-line to help with calculation. They develop ways of recording calculations using pictorial representations. Children should be introduced to the language of subtraction as the difference in a range of practical situations, including using daily routines as a context for learning. For example, comparing the blocks to see how many packed lunches/ school dinners there are on a given day.	  "This tower is 4 bricks more than the 4 brick tower."  "This tower is 4 bricks fewer than the 8 brick tower."
MULTIPLICATION	Children will experience equal groups of objects. They will count in 2s and 10s and begin to count in 5s. They should be provided with practical opportunities and visual images eg: counting pairs of socks or counting in tens to find out how many fingers five children would have. Children should be introduced to the language of doubling in context. E.g. throwing dice, They will work on practical problem solving activities involving equal sets or group	  

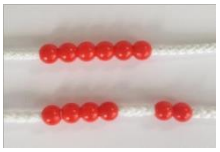
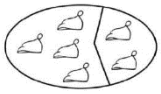
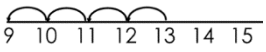
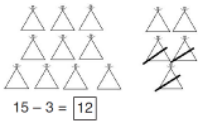
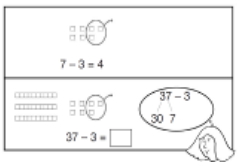
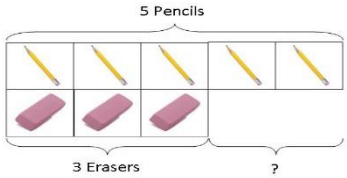
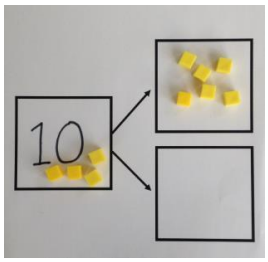
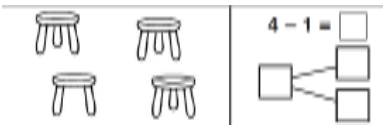
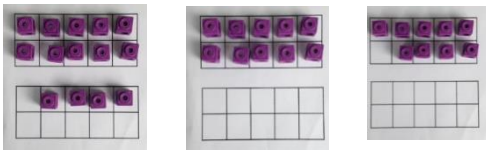
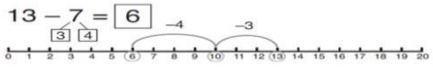
DIVISION Solve problems including halving and sharing	Sharing objects	 One for you. One for me. Is it fair? How many do we each have? 15 shared between 5 
	Grouping objects Put groups onto plates.	How many groups of 4 are there in 12 stars? 

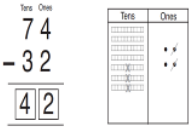
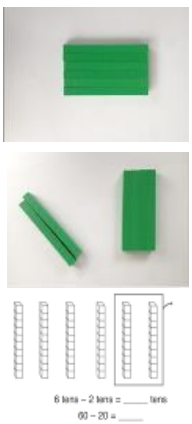
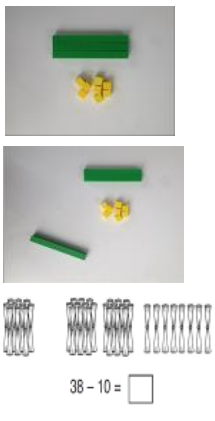
Year 1

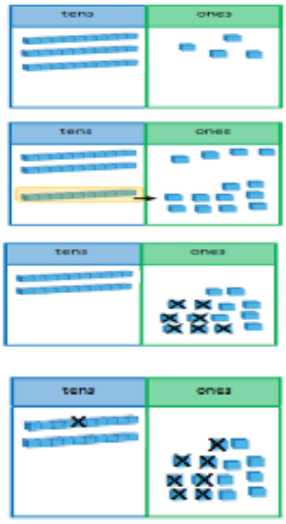
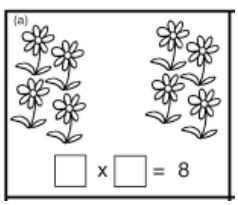
Progression	Notes	Model/ Image
Children can add and subtract one and two digit numbers.	**Non-negotiable: by the end of Year 1, all children must know their +/- facts for all numbers to 10.**	
ADDITION **Joining two groups**	Recounting all objects using one-to-one correspondence	3 + 4 = 7    5 + 3 = 8 
Counting on	Single digit number from a single digit number. Single digit number from a 2-digit number.	8 + 1 = 9   17 = 12 + 5  
Bar Method	Part-Whole Model	There are 3 footballs in the red basket 2 footballs in the blue basket. How many footballs are there altogether? 
Part-part-whole	Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them. Pupils could place ten on top of the whole as well as writing it down. The parts could also be written in alongside the concrete representation.	 10 = 6 + 4 10 - 6 = 4 10 - 4 = 6 10 = 4 + 6 

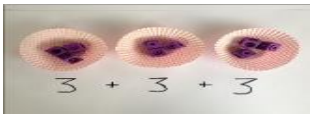
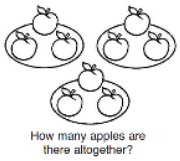
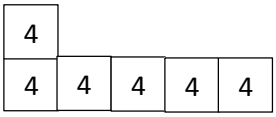
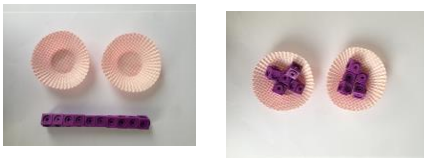
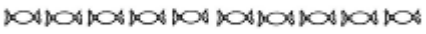
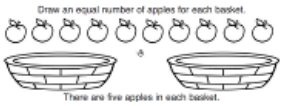
Regrouping ones to make ten	This is an essential skill that will support the make ten strategy and column addition.) The colours of the beads on the bead string make it clear how many more need to be added to make ten	6 + 5 = 11  4 + 9 = 13  
Adding three single digit numbers (make ten first)	Pupils may need to try different combinations before they find the two numbers that make 10. The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten. The second bead string shows 4, 6 and then 7. The final bead string shows how they have now been put together to find the total.	1 more than 5 5 + 1 = 6  2 more than 5 5 + 2 = 7  
Adding 1, 2, 3 more	Here the emphasis should be on the language rather than the strategy. As pupils are using the bead string, ensure that they are explaining using language such as; '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'	 

Column method for addition, no regrouping .	Place value grids and Dienes blocks should be used as shown in the diagram before moving onto the pictorial representations. Dienes blocks should always be available, as the main focus in Year 1 is the concept of place value rather than mastering the procedure.	 Tens Ones 4 2 + 2 6 6 8  Tens Ones 4 2 2 6 6 8
Column method for addition, regrouping .	Dienes blocks and place value grids should be used as shown in the diagrams. Even when working pictorially, pupils should have access to Dienes blocks	 17 + 14 = 31  Tens Ones 5 3 + 8 6 1 1 First add the ones. Re-group 10 ones to 1 ten. Next add the tens.
Adding multiples of ten	Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important, as pupils need to understand that it is a ten and not a one that is being added.	50 = 30 + 20   3 tens + 5 tens = _____ tens 30 + 50 = _____  36 + 40 = ☐

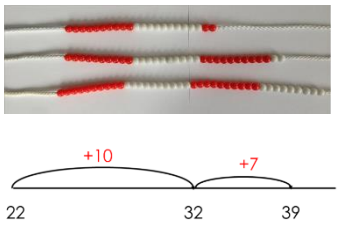
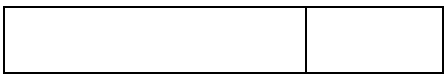
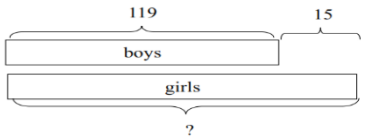
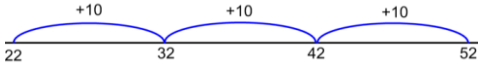
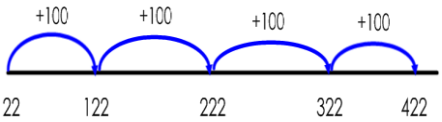
<u>SUBTRACTION</u> **Taking away from the ones**	When this is first introduced, the concrete representation should be based upon the diagram. Real objects should be placed on top of the images as one-to-one correspondence, progressing to representing the group of ten with a tens rod and ones with ones cubes.	    4 = 6 – 2 13 - 4 = 9
Counting back	Single digit number from a single-digit number Single digit number from a 2 digit number	   28 - 4 = 
Bar Method	Comparison Model	Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have? 
Part-part-whole	Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them. Pupils start with ten cubes placed on the whole. They then remove what is being taken away from the whole and place it on one of the parts. The remaining cubes are the other part and also the answer. These can be moved into the second part space.	  14 – 5 =
Make ten strategy	Single digit number from a 2-digit number Pupils identify how many need to be taken away to make ten first. Then they take away the rest to reach the answer.	 

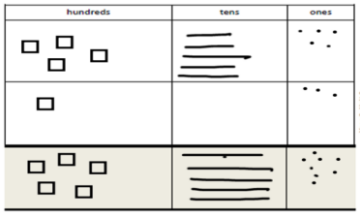
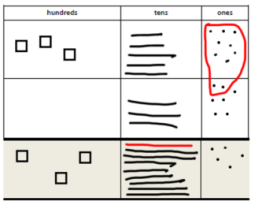
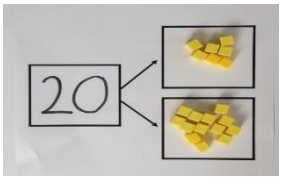
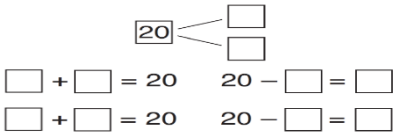
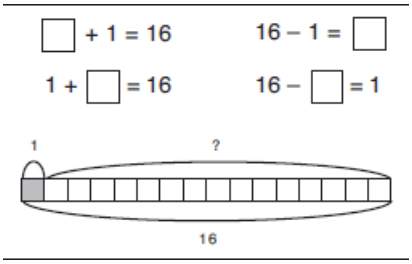
Regroup a ten into 10 ones	After the initial introduction, the Dienes blocks should be placed on a place value chart to support place value understanding. This will then support pupils when using the column method.	  9 = 15 - 6
Taking away from the tens .	Pupils should begin to identify which equations require taking away from the tens and which from the ones	 
Column method without regrouping		34 - 13 = 21  
Subtracting multiples of ten	Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important as pupils need to understand that it is a ten not a one that is being taken away.	40 = 60 – 20  38 - 10 = 28 

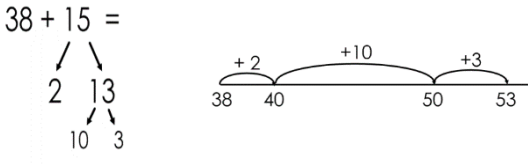
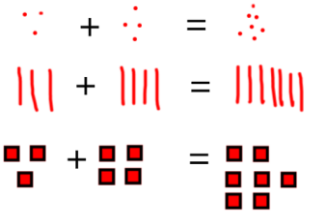
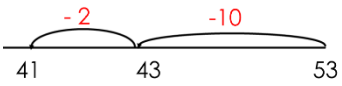
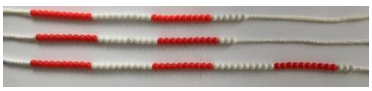
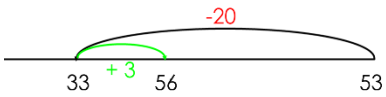
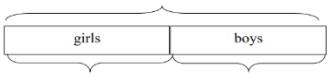
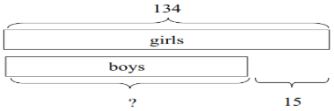
Column method with regrouping	This example shows how pupils should work practically when being introduced to this strategy.	34 - 17 = 17 
<u>MULIPLICATION</u> Skip counting in multiples of 2, 5, 10 from zero	The representation for the amount of groups supports pupils' understanding of the written equation. So two groups of 2 are 2, 4. Or five groups of 2 are 2, 4, 6, 8, 10. Count the groups as pupils are skip counting. Number lines can be used in the same way as the bead string. Pupils can use their fingers as they are skip counting.	 4 × 5 = 20
Making equal groups and counting the total	How this would be represented as an equation will vary. This could be 2 × 4 or 4 × 2. The importance should be placed on the vocabulary used alongside the equation. So this picture could represent 2 groups of 4 or 4 twice.	 2 × 4 = 8   Draw  to show 2 x 3 = 6

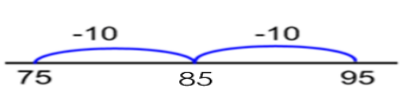
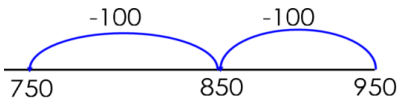
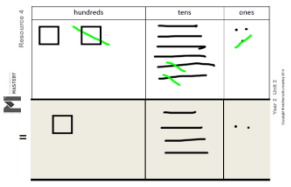
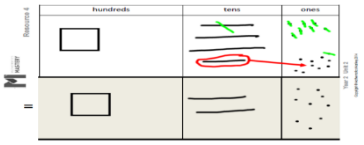
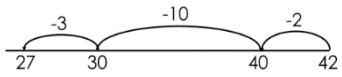
Solve multiplications using repeated addition		  3 + 3 + 3 = 9
Bar Method	Comparison Model	Peter has 4 books Harry has five times as many books as Peter. How many books has Harry? 
<u>DIVISION</u> **Sharing objects into groups**	Pupils should become familiar with division equations through working practically. The division symbol is not formally taught at this stage.	10 ÷ 2 = 5  There are 10 sweets. Ring groups of 2.  There are _____ groups of 2. 

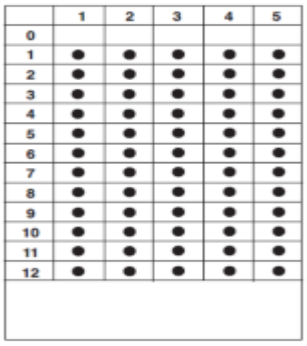
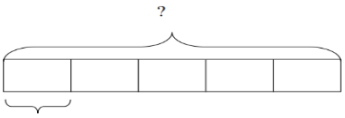
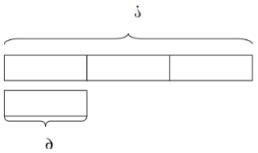
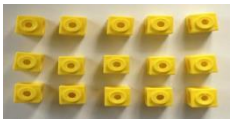
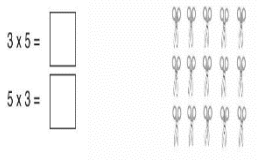
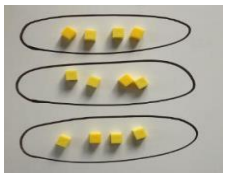
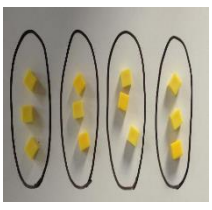
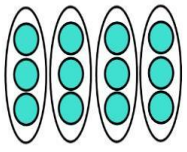
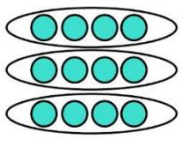
Year 2

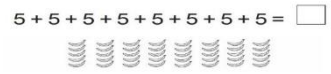
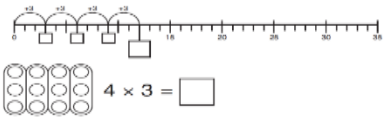
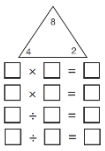
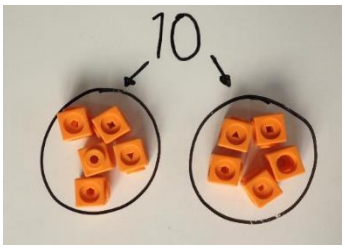
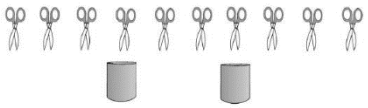
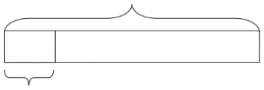
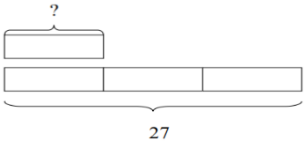
Progression	Notes	Model/ Image
<u>ADDITION</u> **Partitioning one number, then adding tens and ones.**	Pupils can choose themselves which of the numbers they wish to partition. Pupils will begin to see when this method is more efficient than adding tens and taking away the extra ones, as shown	 22 + 17 = 39
Rounding one number, then adding the tens and taking away extra ones.	Pupils will develop a sense of efficiency with this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones.	 22 + 17 = 39
Bar Method	Part-Whole Model	Tom has a bag of 64 marbles, his friend gives him 28 more, how many does he have now? Whole 
	Comparison Model	119 boys took part in an art competition. 15 more girls than boys took part. How many girls took part in the competition? 
Counting on in tens and hundreds	Dienes blocks should be used alongside the pictorial representations; they can be placed on the place value grid before pupils make pictorial representations. As in Year 1, the focus for the column method is to develop a strong understanding of place value.	  

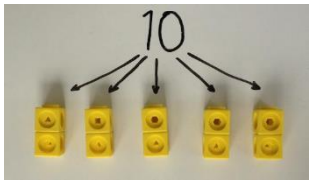
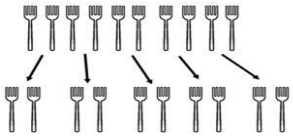
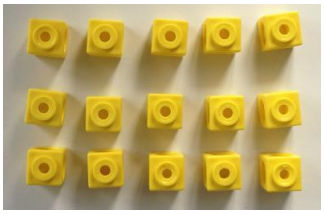
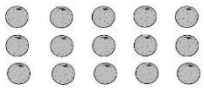
Column method without regrouping	Dienes blocks should be used alongside the pictorial representations; they can be placed on the place value grid before pupils make pictorial representations. As in Year 1, the focus for the column method is to develop a strong understanding of place value.	 **hundreds** | **tens** | **ones** 4 | 5 | 5 1 | 0 | 3 5 | 5 | 8
Column method with regrouping	Pupils explore the different ways of making 20. They can do this with all numbers using the same representations.	 hundreds | tens | ones 3 | 5 | 8 + | 3 (1) | 7 3 | 9 | 5
Part-part-whole	How pupils choose to apply this strategy is up to them; however, the focus should always be on efficiency.	   

Make ten strategy **Using known facts**	Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.	   3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 70
<u>SUBTRACTION</u> **Subtracting tens and ones**	Pupils must be taught to partition the second number for this strategy. Pupils will begin to see when this method is more efficient than subtracting tens and adding the extra ones, as shown.	  53 - 12 = 41
Subtracting tens and adding extra ones	Pupils must be taught to round the number that is being subtracted. Pupils will develop a sense of efficiency with this method, beginning to identify when this method is more efficient than subtracting tens and then ones.	  53 -17 = 36
Bar Method	Part-whole Model	135 children took part in an art competition. There were 62 girls. How many boys were there? 135  62 ?
	Comparison Model	134 childrren took part in an art competition. 15 fewer boys than girls took part. How many boys took part in the competition. 

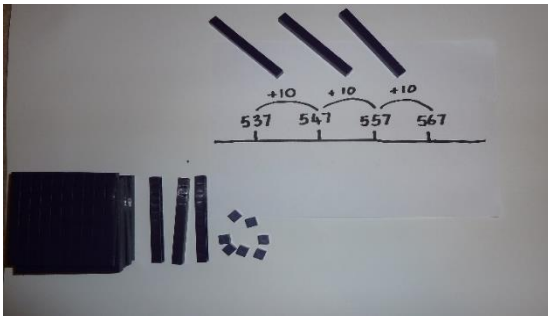
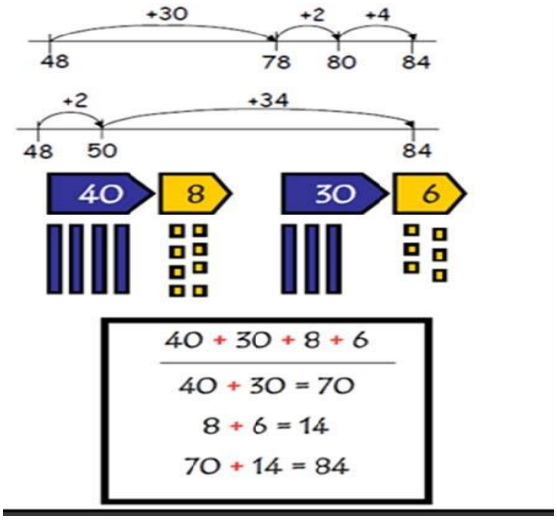
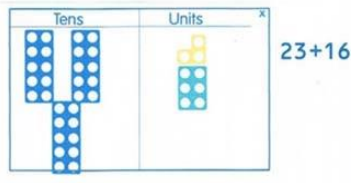
Counting back in multiples of ten and one hundred	As in Year 1, the focus for the column method is to develop a strong understanding of place value and pupils should always be using concrete manipulatives alongside the pictorial.	  
Column method without regrouping	As in Year 1, the focus for the column method is to develop a strong understanding of place value and pupils should always be using concrete manipulatives alongside the pictorial.	 263 - 121= 142
Column method with regrouping	How pupils choose to apply this strategy is up to them. The focus should always be on efficiency.	 147 – 18 = 129
Bridging through ten	Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.	 42 - 15 = 2 13 10 3 
Using known number facts		 8 - 4 = 4 *leads to* 80 - 40 = 40 *Leads to* 800 - 400 = 400

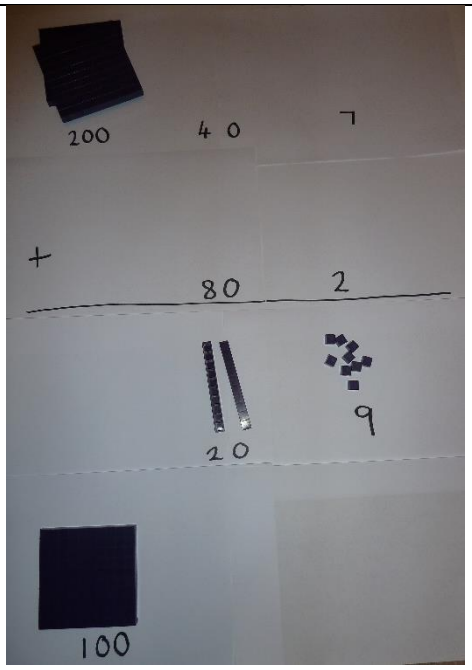
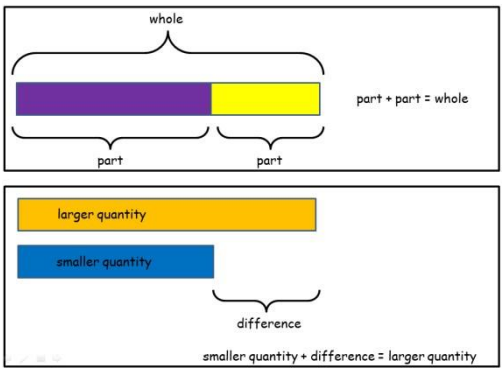
MULTIPLICATION **Skip counting in multiples of 2, 3, 4, 5, 10 from 0**	Pupils can use their fingers as they are skip counting, to develop an understanding of 'groups of'. Dotted paper is used to create a visual representation for the different multiplication facts. Each multiplication table has its own layout.	 
Bar Method	Part-Whole Model	Devi saved £8 a week for 5 weeks. How much did she save altogether?  £8
	Comparison model	There are 9 white flowers. There are 3 times as many red flowers as white flowers. How many red flowers are there? 
Multiplication is commutative	Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	      12 = 3 × 4 12 = 4 × 3

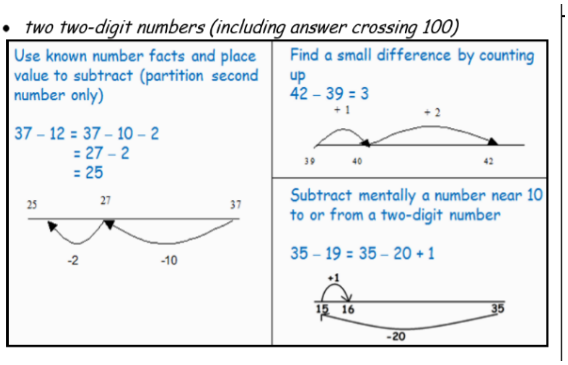
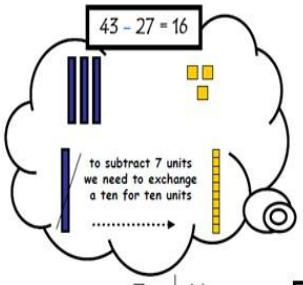
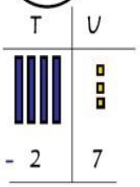
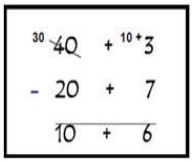
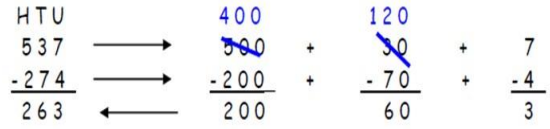
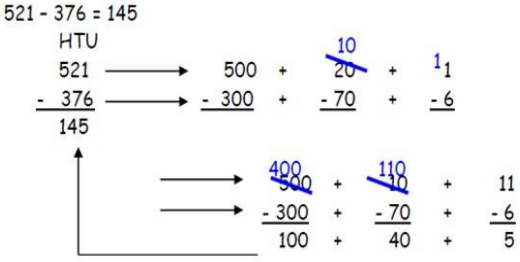
Multiplication as repeated addition	Pupils will apply skip counting to help find the totals of these repeated additions.	   
Using the inverse	This should be taught alongside division, so pupils learn how they work alongside each other.	 
<u>DIVISION</u> **Division as sharing**	Here, division is shown as sharing. If we have ten pairs of scissors and we share them between two pots, there will be 5 pairs of scissors in each pot.	 10 ÷ 2 = 5 
Bar Method	Part-Whole Model	Devi has £8 each week. How many weeks will it take her to save £40? £40  £8
	Comparison Model	There are 27 red flowers. There are three times as many red flowers as white flowers. How amny white flowers are there? 

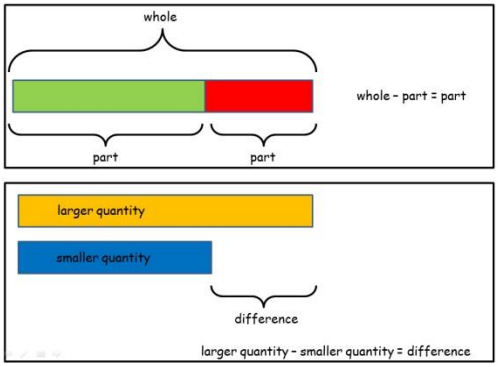
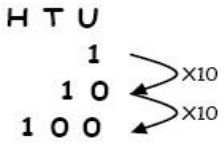
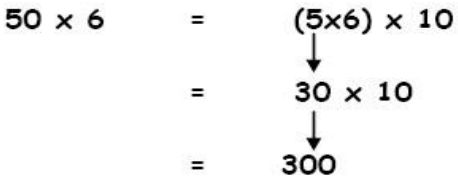
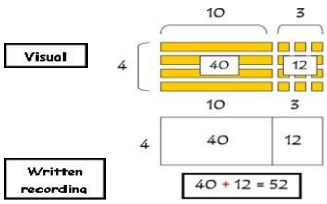
Division as grouping	Here, division is shown as grouping. If we have ten forks and we put them into groups of two, there are 5 groups.	10 ÷ 2 = 5  
Using the inverse	This should be taught alongside multiplication so that pupils learn how they work alongside each other.	 15 ÷ 5 = 3 15 ÷ 3 = 5 

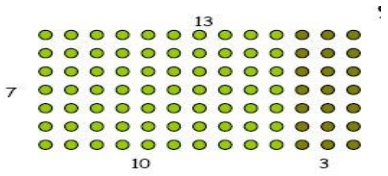
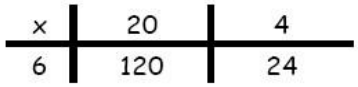
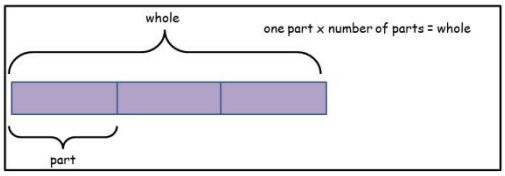
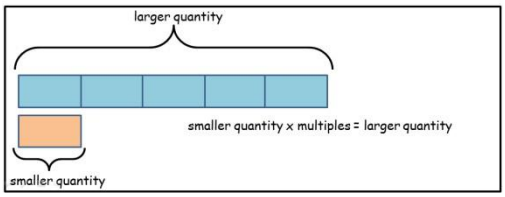
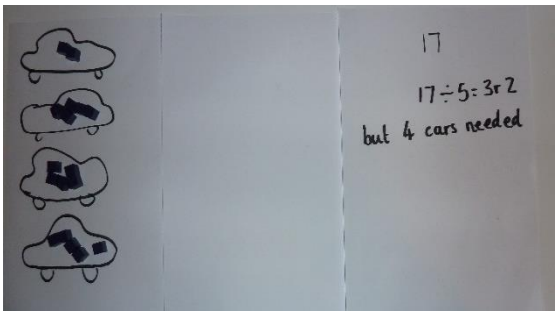
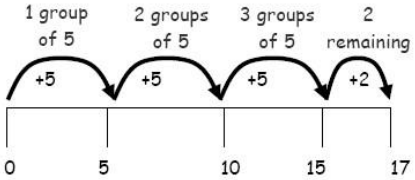
Year 3

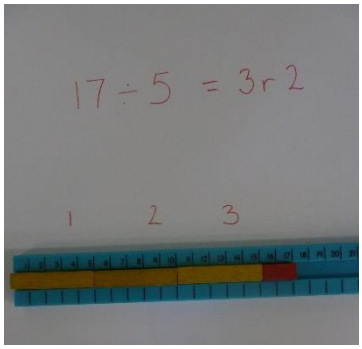
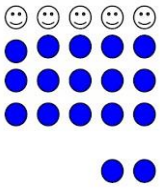
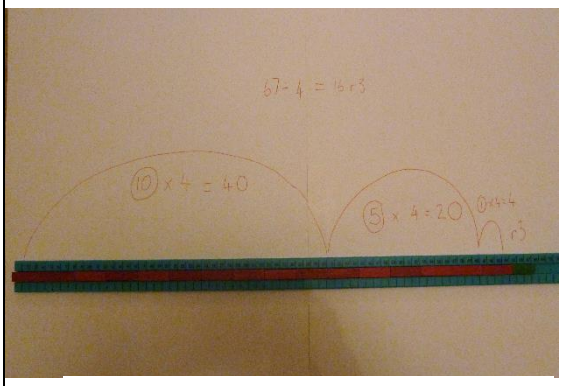
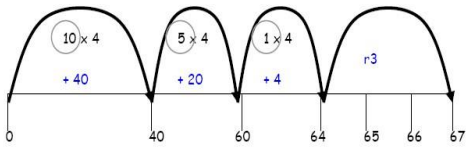
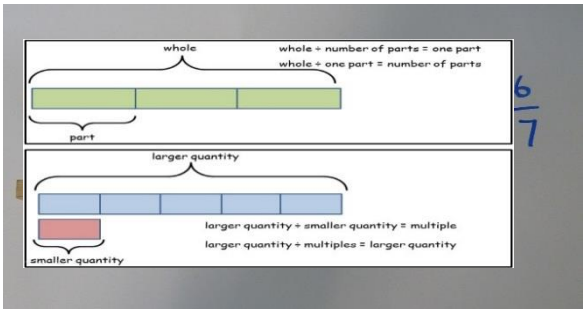
Progression	Notes	Models/images
ADDITION Mentally add a three-digit number and 1s and 10s and 100s. **Counting on**	Children will visualise counting on in 1s, tens and hundreds, initially supported by jottings.	Blank number lines. Go back to diennes where necessary.  365 + 90, 365 + 100 = 465, 465– 10 = 455
Add to near numbers and adjust Mentally add two, two-digit numbers. **Counting on using a number line** **Partition and recombine two, two digit numbers.**	Children will develop fewer more efficient steps. Partitioning is a mental method but it is useful for children to record jottings. Children need to add two-digit numbers that bridge through the tens boundary.	  TU 23 → 20 + 3 + 16 → 10 + 6 39 ← 30 + 9

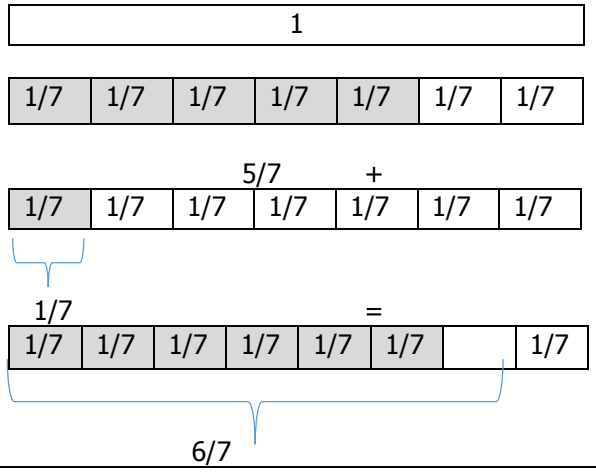
Add numbers with up to 3 digits using formal written methods of column addition **The expanded column method.** Solve problems, including missing number problems, using number facts, place value, and more complex addition **Bar method**	The expanded method allows children to see what happens to numbers in the standard written method. Problems and calculations should involve missing numbers and operations in all positions, including moving the equals sign 78 = 67 + ☐	 HTU 247 → 200 + 40 + 7 + 82 → 80 + 2 329 ← 300 + 20 + 9 100 Diennes to practice exchanging 1s and tens and tens and 100s. Arrow cards to support partitioning numbers. 
<u>SUBTRACTION</u> Mentally subtract from a three-digit number, 1s, 10s and 100s. **Counting back** **Counting on Adding near numbers and adjusting.**	Children visual counting back in 1s, 10s and 100s, initially supported by number lines. Counting on where the difference is small.	Blank number lines, to count back in jumps of 10 and 100. 678 – 90 = 678 -100 + 10

Mentally subtract two, two digit numbers Subtract numbers with up to 3 digits using formal written methods. **Expanded column method.**	This is an important method that should not be skipped as it help children understand the process of subtraction (particularly the 'exchanging' element). They will use the expanded column method (known as decomposition) to partition the number and subtract each place value separately always starting with the least significant digits (e.g. the units). It is very important they understand the importance of keeping the digits lined up. First they will work with numbers that have no exchanging (e.g. the units or tens of the number being subtracted is smaller than the starting number). **<u>Exchanging the tens to units</u>** Next they will learn how to exchange from the tens to the units. They need to recognise when the starting number's units have less than the number being subtracted. When this is the case they need to **'exchange'** 10 from the tens into the units, in order to be able to subtract the numbers. It	• *two two-digit numbers (including answer crossing 100)*   Expanded method It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.    <u>Exchanging tens to units and hundreds to tens</u>  Once the children understand this process of exchanging, they will often do it in one go to save time. Arrow cards to support partitioning numbers.

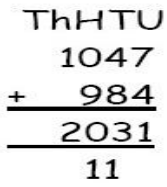
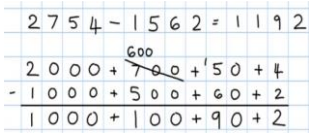
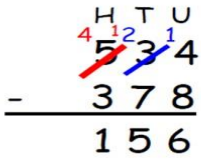
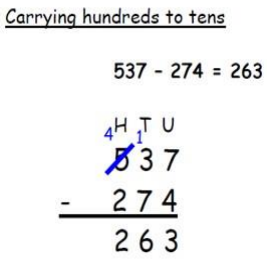
Solve problems, including missing number problems, using number facts, place value, and more complex subtraction. **Bar method**	is important the children understand the tens have to come over to the units as a whole 10. The Base 10 (Dienes) apparatus helps the children to visualise what happens and how the exchange takes place. **<u>Exchanging from hundreds to tens</u>** Introduced next and again the children need to understand that 100s are exchanged as a whole 100. (A common misconception is that they try to just take 10 from the 100s). The Base 10 (Dienes) apparatus helps the children to visualise what happens and how the exchange takes place. Problems and calculations should involve missing numbers and operations in all positions, including moving the equals sign 78 = □ - 13	
<u>MULTIPLICATION</u> Children write and calculate mathematical statements for multiplication using the multiplication tables that they know. **Use place value to multiply by multiples of 10.** Multiply two-digit numbers by one digit numbers, using mental methods, progressing to formal written methods. **Partitioning**	They will understand when you multiply by 10 the number gets 10 times greater and moves one place left on the Place Value Grid. They will use known multiplication facts to derive related facts. They will begin by partitioning the two digit number into tens and units and then multiply each part separately. Then they will add all the answers together to get the final product. Using the Deines to show	 Place value charts Dienes  

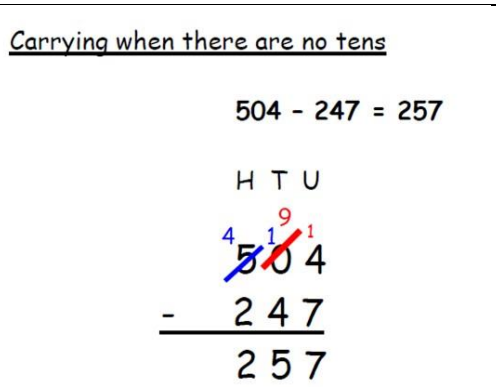
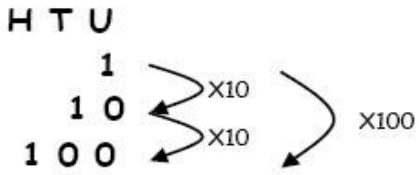
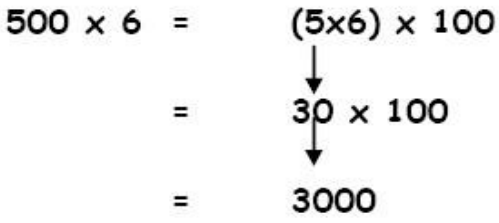
Grid method Solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects **Bar method**	how this works visually or by splitting an array of 13x7 into 10x7 and 3x7 will help the children understand that they can do this and still have the same answer. They will be confident at applying their mental times-table knowledge to simplify numbers e.g. thinking of 20x6 as 2x6 and then making their answer ten-times bigger. They will understand the roll of the place holder to make numbers ten times smaller or larger. Problems and calculations should involve missing numbers and operations in all positions, including moving the equals sign. 12 x 3 = ☐ ☐ x 6 = 78	 24 x 6 = 144  H T U 1 2 0 + 2 4 1 4 4  
DIVISION Divide two-digit numbers by one digit numbers, using mental methods, progressing to formal written methods. Repeated addition, including finding remainders. **Grouping**	The children will use number lines in the same way as before, with repeated addition, but they need to recognise if they cannot group anymore what-ever sized jump needed to get to the number being divided is the remainder.	A car holds 5 people, how many cars are needed to transport 17 people?  

	The cuisinaire rods and Numicon number rod tracks can be used as the concrete version of this if pupils need to refer back to the concrete.	 Cuisinaire and Numicon number rod trackes
Arrays	The children will need to have experience of questions like these, where they have to think about how to deal with the remainder answer. They need to understand how to put the answer back into the context of the question.	Sharing problems I share 17 lollies equally between 5 party bags, how many lollies are in each bag? 
Chunking along the number line Solve problems, including missing number problems, involving division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects **Bar Method** Add and subtract fractions with the same denominator within one whole. **Adding parts of a whole** **Bar method**	When the children move to dividing larger two-digit numbers they will use repeated addition, however they will apply their times table knowledge and count on in 'chunks'. This makes the number line shorter and therefore the workings quicker to solve. They must circle the 'chunks' and then add them up mentally. Problems and calculations should involve missing numbers and operations in all positions, including moving the equals sign. 17 ÷ ☐ = 3 r2	 67÷4 = 16 r 3  10 + 5 + 1 = 16 remainder 3 

	Children will recognise the denominator as the number the whole is divided by and use this in helping to choose Cuisenaire rods to use e.g. a whole of 7 is divided into parts of 1.	

Year 4

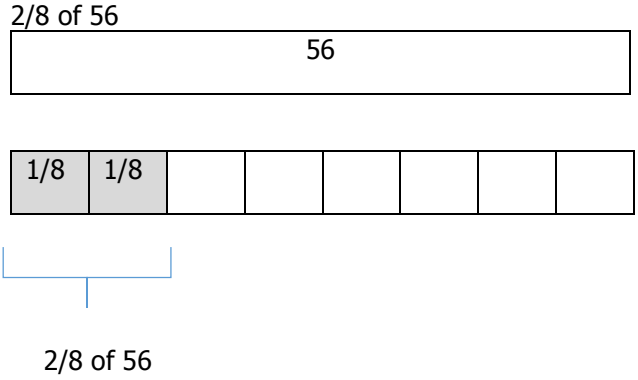
Progression	Notes	Models and Images
ADDITION Children will be able to add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate. **Counting on** **Partitioning and recombining** **Compact Method**	Children will visualise counting on in jumps of 1s, 10s, 100s from numbers with up to 4 digits. They understand the place value of each digit, adding from the least significant digit first. They can 'carry' below the line when the digits go over their value. Dienes are used to practise exchanging 1s to 10s, 10s to 100s and 100s to 1000s.	Blank number lines. Place value charts. Dienes Jottings  Dienes, arrow cards, place value charts
SUBTRACION Children will be able to subtract numbers with up to 4 digits using the formal written methods of columnar addition where appropriate. **Expanded decomposition method** **Compact Decomposition Method**	They know to look carefully at the numbers and identify which parts (HTU) will need exchanging, before beginning. Often a common misconception is that 10 can be carried from 100 into the units, or 100 can be carried into the units. It is really important the children	 Diennes, arrow cards, place value charts **Carrying tens to units and hundreds to tens.**  

	understand they have to first carry 100 to the tens and then carry ten into the units – so effectively it takes two steps to get the units ready for the subtraction to take place. **Using the Base 10 (Dienes) to show this visually will help them to understand this.** and understand the importance of lining up each digit. They understand the place value of each digit, subtracting from the least significant digit first. They know to look carefully at the numbers and identify which parts (HTU) will need exchanging, before beginning. They still deal with the least significant digit first.	
MULTIPLICTAION The children will be able to multiply any whole number by 10 or a multiple of 10, 100 or a multiple of 100. Multiply two-digit and three-digit numbers by a one-digit number using formal written layout. **Grid method**	They will understand how to move the digits over 1 place to x10 and two places to x100. They will be very secure with place value and be confident to manipulate numbers in questions like 40x50 to simplify to 4x5 and then understand they made both numbers ten times smaller, so their answer will need to be made ten-times bigger twice (one hundred-times bigger). Diennes are used to model 7 lots of 200.	  231 x 7 = 1617 Diennes

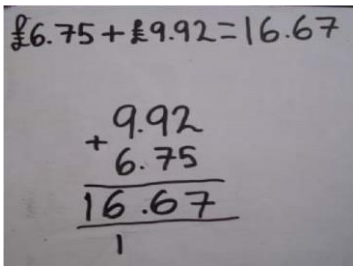
×	7
200	1400
30	210
1	7
	1617

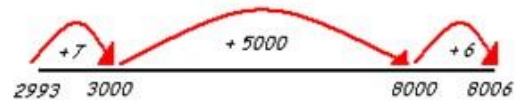
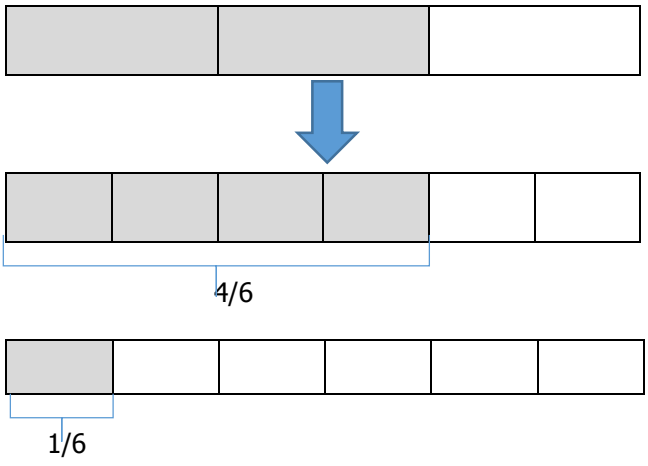
+

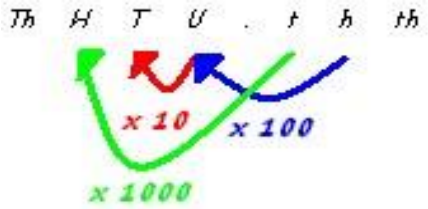
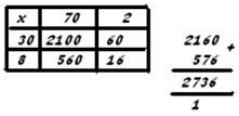
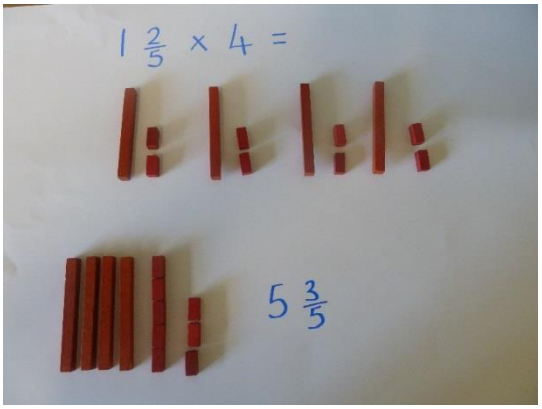
Short multiplication	When children are secure they can move from the grid method to short multiplication with 2 and 3 digit numbers and a 1 digit number.	452 X 3 1356 1
DIVISION Divide numbers with up to three digits. **Chunking method, progressing to more efficient chunking.** Solve problems involving increasingly harder fractions to calculate	The children will use repeated addition and chunking to solve larger divisions more efficiently; they will apply their times-table knowledge, beginning with zero and adding 'chunks' until they get to the required number or have a remainder. (As they become more confident with the method they will lose writing down the beginning zero, as they realize they don't need to record it, but to begin with it is useful to remind them of their starting point.) Key facts are times-table facts for the divisor that they should note down before beginning – so they have all the information they need to hand. Again they need to understand it is the 'chunks' that give them the answer. Circle the 'chunks' and add up mentally: 10 + 5 + 1 = 16 and remember to include the remainder with the answer. Circle the 'chunks' and add up mentally: 20 + 20 + 3 = 43.	**63 ÷ 3 = 21** H T U 0 + 30 (10 x 3) 30 + 30 (10 x 3) 60 + 3 (1 x 3) **63** Circle the 'chunks' and add up mentally: 10 + 10 + 1 = 21 KEY FACTS: 100 x 3 = 300 10 x 3 = 30 5 x 3 = 15 2 x 3 = 6 1 x 3 = 3 **67 ÷ 4 = 16 r 3** H T U 40 (10 x 4) + 20 (5 x 4) 60 + 4 (1 x 4) 64 + 3 (remainder) **67** KEY FACTS: 100 x 4 = 400 10 x 4 = 40 5 x 4 = 20 2 x 4 = 8 1 x 4 = 4 **258 ÷ 6 = 43** H T U 120 (20 x 6) + 120 (20 x 6) 240 + 12 (3 x 6) 252 + 6 (1 x 6) **258** KEY FACTS: 100 x 6 = 600 50 x 6 = 350 20 x 6 = 120 10 x 6 = 60 5 x 6 = 30 2 x 6 = 12 1 x 6 = 6

quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number. **Bar method** **Chunking**	Making the relationship between an eighth and dividing a whole by 8, we can chunking along the number line in 8 to 56 and determine the quantity of 1/8 and doubling to make 2/8.	2/8 of 56  2/8 of 56 Cuisenaire and number rod track. 0 – 100 number track

Year 5

Progression	Notes	Models/images
<u>ADDITION</u> Add numbers mentally with increasingly large numbers. **Partition and recombine.**		Blank number lines Partition both numbers and recombine 2358 + 773 = 2000 + 300 + 50 + 8 + 700 +70 + 3 = 2000 + 1000 + 120 + 11 = 3000 + 100 + 30 + 1 =3131
Partitioning with a number line.	Children will identify the larger quantity, the number with most digits, to place on a number line. They will identify the smaller quantity, the number with fewer digits to partition and add.	Partitioning with number lines 
Partition smaller quantity only. **Add the nearest multiple of 10 or 100, then adjust.**	Children will identify the smaller quantity, the number with fewer digits to partition and add. Jottings will support this.	Partition second number only into hundreds, tens and ones and recombine 2358 + 773 = 2358 + 700 + 70 + 3 = 3058 + 70 + 3 = 3128 + 3 = 3131 Add the nearest multiple of 10 or 100, then adjust 458 + 79 = 458 + 80 - 1
Add numbers with more than 4 digits, including using formal written methods. **Compact Addition**	Using place value charts (which include 8 place values including 2 decimal places) to partition numbers, whilst recognising that the pattern of place value continues. They apply the same rules with 'carrying' to decimal numbers. Using pennies to exchange for 10 pence and ten pence to exchange a pound.	Place value charts Pennies, ten pence, pounds. TU . $\frac{1}{10}$ 24.6 + 34.7 59.3 1  TU . $\frac{1}{10}$ $\frac{1}{100}$ 24.6 4 + 34.7 9 59.4 3 1 1
<u>SUBTRACTION</u> Subtract numbers mentally with increasingly large numbers. **Subtract the nearest multiple of 10 or 100, then adjust.**		

Counting on Subtract numbers with more than 4 digits, including using formal written methods. **Compact Addition**	They apply the same rules with 'exchanging' to decimal numbers. They partition numbers using place value chart, including numbers with zero as a place holder. They partition numbers with 2 and 3 decimal places using place value charts.	Subtract the nearest multiple of 10 or 100, then adjust 458 - 79 = 458 - 80 + 1 = 378 + 1 = 379 Find a difference by counting up 8006 - 2993 = 5013  38.2 - 24.7 = 13.5 T U 1/10 37 8 . 1 2 - 2 4 . 7 1 3 . 5 64.21 - 21.72 = 42.49 T U 1/10 1/100 6 3 4 . 1 2 1 - 2 1 . 7 2 4 2 . 4 9
ADDITION & SUBTRACTION Add and subtract fractions with the same denominator, and denominators that are multiples of the same number. **Bar Method**	This can be paralleled with Cuisenaire rods to revert back to the concrete.	2/3 + 1/6 = 5/6 

MULTIPLICATAION multiply whole numbers and those involving decimals by 10, 100 and 1000	They will understand the place value of decimal numbers and know how to move the digits over 1 place to the left to x10, two places to x100, and three places to x1000.	
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers. **Grid method**	Multiply 2 two-digit numbers using the grid method. Children derive multiplication facts from their knowledge of times tables e.g. 70 x 30 = 2100, using 7 x 3 =21 and 10 x 10 = 100.	Dienes Place value charts Arrow cards Start with grid method when multiplying by 2 digit numbers 72 x 38 is approximately 70 x 40 = 2800 
Long multiplication, Expanded method.		35 x 37 = 1295 Th H T U 3 5 x 3 7 3 5 (7 x 5) 2 1 0 (7 x 30) 1 5 0 (30 x 5) + 9 0 0 (30 x 30) 1 2 9 5 1
Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams. **Arrays**	Children will understand the use of the denominator as the number of parts in the whole and use in selecting the Cuisenaire rods. They will create arrays of the whole and parts. They will count the wholes together and will	

<table>
<tr>
<td>Bar model</td>
<td>'exchange' parts for whole where possible to create an answer with a proper fraction.

Partition whole numbers and fractions to multiply whole numbers mentally and use the bar method to multiply the fraction.</td>
<td>1 2/5 x 4 = (1 x 4) + (2/5 x 4) = 4 + 1 3/5 = 5 3/5
<table>
<tr><td>2/5</td></tr>
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<table>
<tr><td>2/5</td><td>2/5</td><td>2/5</td><td>2/5</td></tr>
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8/5 or 1 3/5</td>
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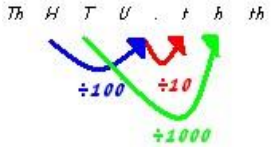
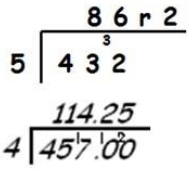
<tr>
<td><u>DIVISION</u>
Divide whole numbers and those involving decimals by 10, 100 and 1000
Place value

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
Short division

Chunking, using known number facts</td>
<td></td>
<td> 

Place value charts including decimal places

Bus shelter method (short division)
 

Pupils should consider whether remainders should be left as a reminder, rounded to the nearest whole or converted into a decimal or fraction.

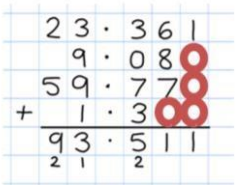
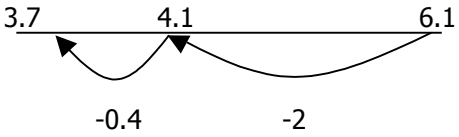
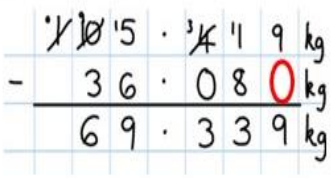
Introduce long division (dividing by single digits)

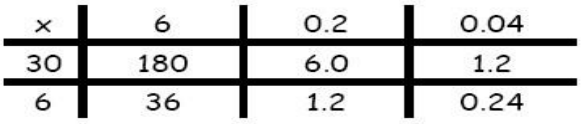
256 ÷ 7 lies between 210 ÷ 7 = 30 and 280 ÷ 7 = 40

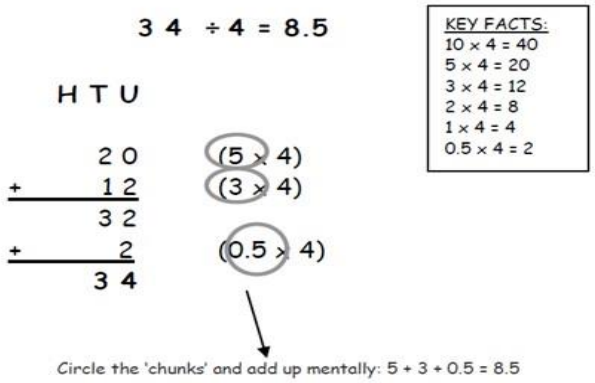
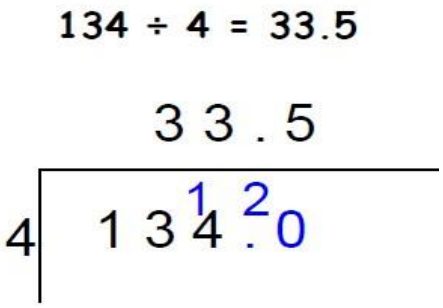
256
- 70 (10 groups) or (10 x 7)
186
140 (20 groups) or (20 x 7)
46
- 42 (6 groups) or (6 x 7)
4 (36 groups) or (36)

Answer: 36 remainder 4</td>
</tr>
</table>

Year 6

Progression	Notes	Models/Images
ADDITION Perform mental calculations, including with mixed operations and large numbers (and decimal numbers) **Partition and recombine**	They partition 7 and 8 digit numbers using place value chart, including numbers with zero as a place holder. They partition numbers with 2 and 3 decimal places using place value charts. Children will progress from partitioning both numbers to only partitioning the smaller quantity.	Place value charts Partition both numbers into hundreds, tens, ones and decimal fractions and recombine 35.8 + 7.3 = 30 + 5 + 0.8 + 7 + 0.3 = 30 + 12 + 1.1 = 42 + 1.1 = 43.1 Partition second number only into hundreds, tens, ones and decimal fractions and recombine 35.8 + 7.3 = 35.8 + 7 + 0.3 = 42.8 + 0.3 = 43.1
Adding numbers involving mixed decimals using the **Compact column method**	Children should be reminded of the importance of aligning the columns accurately. Where there is an 'empty' space in a decimal column, pupils could insert a zero to show the value.	
SUBTRACTION Perform mental calculations, including with mixed operations and large numbers (and decimal numbers) **Counting back**		Use known number facts and place value to subtract 6.1 – 2.4 = 3.7 
Subtracting numbers involving decimals using the **compact column method**	Apply using the column method to problem solving involving money and measures. Align the decimal point when setting out the calculation. Use place holders to aid the understanding of the value in that column	

<u>MULTIPLICATION</u> Multiply one-digit numbers with up to 2 decimal places by whole numbers	They will understand the place value of decimal numbers and will confidently be able to multiply decimals in the context of measures or money.	6.24 x 36 = 224.64 
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of **long multiplication**	Set out in the traditional column way making sure the HTU are lined up, however breaking down each part of the process separately (as you would in the Grid Method, linking this to the grid method to show correlating calculations) and showing the multiplication for each step in brackets. **1. Always begin with the least significant digit, which in this example is the units: 7x5=35, lining up the answer in the correct columns.** **2. Next 7x30=210, this is written underneath the previous answer.** **3. Now on the line below, 30x5 is going to be solved.** **4. Then you need to solve 30x30.** **5. Finally add all the parts together, as you would in the column addition method, starting with the least significant digit first.** Set out in the traditional column way making sure the HTU are lined up. **6. Begin with the units 7x5=35, so the 5 goes under the units and the 3 (30)is carried to the tens.** **7. Next 7x30=210, but remember to add the 30 you carried onto this total.** **This answer is written on the same line as the first answer, because this is all part of the 35x7.** **8. Now on the line below, 35x30 is going to be solved.** **9. You need to solve 30x5=150. The 0 (0) goes under the units column, the 5 (50) goes under the tens column and the 1 (100) is carried over to the hundreds.**	35 x 37 = 1295 Th H T U 3 5 x 3 7 3 2 4 5 1 1 0 5 0 1 2 9 5

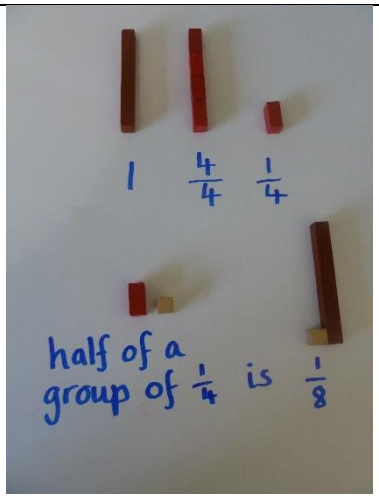
	10. Next solve 30x30=900. The 9 (900) goes under the hundreds column, but remember to add the carried hundreds onto it. **11. Finally add the answers together, as you would in the column addition method, starting with the least significant digit first**	
<u>DIVISION</u> Use written division methods in cases where the answer has up to 2 decimal places **Chunking using decimal chunks to divide remainders.**	The children will use repeated addition and chunking to solve divisions that have a remainder and then deal with the remainder by dividing by decimals.	
Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context **The Compact Method**	1. Set out the compact method with the divisor on the outside and the number being divided (the dividend) in the middle. 2. Start by looking at the most significant number and work out how many times it can go into the number. 3. Now work out how many fours 'go into' 14. Again the answer is 3, this is recorded above. However, there are still 2 'left over'. 4. With the 2 'left over' we still need to divide further and a decimal point is added to the number with a place holding zero. The 'left over' 2 can now be carried to the place holder – giving us 20. 5. Finally we have a number that is divisible by 4 and the final answer of 5 is put above, remembering to bring the decimal point up in front of it	

FRACTIONS

Multiply simple pairs of proper fractions, writing the answer in its simplest form.

Parts of whole

The vocabulary of the operation is important when multiplying by a fraction. One group of ¼ is ¼, half a group of ¼ is 1/8.
Children will understand the use of the denominator as the number of parts in the whole and use in selecting the Cuisenaire rods.



Bar Method

¼	¼	¼	¼

Half a group of ¼

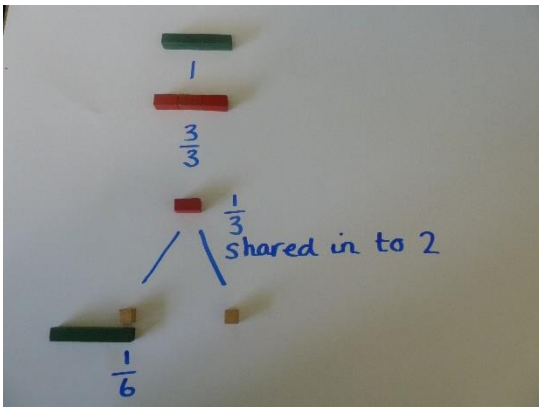
1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

½ x ¼ or ¼ x ½

Divide proper fractions by whole numbers.

Parts of a whole

The vocabulary of the operation is important when dividing by a fraction. 1/3 ÷ 2 is 1/3 shared in to 2 groups.
Children will understand the use of the denominator as the number of parts in the whole and use in selecting the Cuisenaire rods.



Bar Method

1/3	1/3	1/3

1/3 shared in to 2

1/6	1/6	1/6	1/6	1/6	1/6